Basics of Aerodynamics in Extreme Ground Effect

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ABSTRACT

This lecture discusses some mathematical models of high speed wing-in-ground effect marine vehicles operating in close proximity of an underlying surface. Mathematics associated with extreme ground effect is shown to be considerably simplified, so that the description of the flow past a lifting surface of infinite aspect ratio can be reduced to 2-D or even 1-D differential equations. The relevant mathematical models yield closed-form solutions useful in design of vehicles of new generations.

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– Asymptotic Methods in Aero-Hydrodynamics of Wings
– Advanced Marine Vehicles
– Wing-in-Ground-Effect Vehicles
– Aerodynamics of Extreme Ground Effect

Three books :

– The Method of Matched Asymptotic Expansions in Hydrodynamics of Wings (Sudostroenie Publishers, Leningrad, 1979)
– Aero-Hydrodynamics of Ships with Dynamic Principles of Support (Text-Book, Sudostroenie Publishers, Leningrad, 1991, Co-Authored by V.K. Treshkov and N.B. Plissov)
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– In 1986-1991 Member of High-Speed Marine Vehicles Committee of the International Towing Tank Conference

INTRODUCTION

At present we are witnessing a remarkable growth of number of high-speed marine vehicles designed and built to meet both passenger and cargo carrying needs of the world. One of the most peculiar and promising of them, in the author's opinion, is ekranoplan, see [1,2,3] (wing-in-ground-effect craft) which takes advantage of favorable influence of the closeness of the underlying surface upon its aerodynamic and moves at aviation speeds at sea level. These "half ships half planes" become all the more efficient for very small ratios of the clearance to the chord. With account of additional requirements of high seaworthiness at take-off and low empty weight fraction the ocean-going ekranoplans should be sufficiently large, so that, in relative terms, they are expected to operate in extreme proximity to water surface. Hence, the interest to the mathematical models of aerodynamics of such craft in extreme ground effect. In what follows we shall discuss some simplifications of the flow description arising from relative closeness of the underlying surface and give some examples associated with practical applications. It turns out in particular, that for a lifting surface of finite aspect ratio the limiting description of the flow at $h \to 0$ (h is a clearance-to-chord ratio) becomes "almost" two-dimensional in the plane parallel to the ground. Thus, the theory of extreme ground effect represents an interesting complement to the Jones and Prandtl theories which reduce the flow problems to the longitudinal and lateral planes correspondingly. Associated mathematics becomes still simpler for a wing which not only moves near the ground, but also has very small gaps under the tips of the endplates. In the latter case the flow description reduces to one dimension, retaining nonetheless the inherent nonlinearity of ground effect phenomena.

SHORT SURVEY ON APPLICATION OF ASYMPTOTICS

The ground effect is most pronounced at small relative clearances h (ratio of the ground clearance to the chord of the wing). So, solutions of the relevant flow problem can be sought in form of an asymptotic expansion with respect to a small parameter related to h. Keldysh and Lavrent'ev [4] applied the parameter $1/h << 1$ to treat the flow past a hydrofoil with submergence exceeding its chord. Similar expansion was used by Plotkin and Kennel [5], Plotkin and Dodbele [6], Plotkin and Tan [7] to solve flow problems for wings and bodies near a solid wall. To ensure better convergence of the solution series at clearances less than the chord, Panchenkov [8] used a parameter $\tau = \sqrt{1+4h^2} - 2h$ to treat the flow problems for a wing moving near the interface. For very large clearances $\tau \sim 1/4h$, so that the asymptotic expansion in τ becomes equivalent to that of [1]. For $h \to 0$, $\tau \to 1$ and the τ expansion loses validity. Experiments show that the maximum aerodynamic efficiency of wing-in-ground-effect vehicles takes places at distances essentially less than the chord and/or the span. Therefore, it is challenging to expand the flow problem solution, starting from $h = 0$ rather than from $h = \infty$. However, a "pedestrian"(straightforward) asymptotic expansion of the solution for $h \to 0$ results in a degeneracy of the flow problem. In this limit, the *channel* between the lifting surface and the ground vanishes. Consequently, it becomes impossible to satisfy the flow tangency conditions on the lower side of the wing and the part of the ground under the wing. If the vertical coordinate is appropriately stretched in order to see what happens in the channel when h goes to zero, the governing Laplace equation loses one dimension (3-D becomes 2-D and 2-D becomes 1-D). Thus, we deal with a singular perturbation problem, for which neither *outer*, nor *inner* is uniformly valid throughout the flow field, see Van-Dyke [9]. The flow problem for a wing in the ground effect features a "coexistence" of the two characteristic length scales (on one hand - the clearance, on another hand, the chord or the span of the wing), the ratio of which is vanishing as h goes to zero. Such a problem can be handled by the method of matched asymptotic expansions (MAE).

Back in 1962 Strand, Royce and Fujita [10] pointed out the channel flow nature of a highly constrained flow between the wing and the ground. They indicated that in the two-dimensional case the channel flow becomes one-dimensional. However, no method was presented to determine the flow rate under the wing without solving the entire flow problem. The importance of the flow in the channel was underlined in a "ram ground effect theory" developed by Lippisch and Colton [11], [12], [13] who focused their effort on calculation on variation of pressure on the lower side of the airfoil operating near the ground. They developed a simple one-dimensional *hydraulic* theory of a wing section in ground effect combining the Bernoulli and the continuity equations in the channel flow and accounting for the influence of viscosity on the pressure distribution through boundary layer corrections. Gallington *et al.* [11] considered the influence of gaps under the tips of the endplates on an additional simplifying assumption that the pressure in the channel under the wing is constant chordwise. The ensuing *slit-orifice* theory revealed some important similarity factors later validated in the model tests. The first MAE applications for steady lifting flows near the ground are due to Widnall and Barrows [15]. They used a linear theory, assuming that deflections of the (thin) wing surface from corresponding horizontal planes was small compared to characteristic ground clearance. It was shown, in particular, that while the perturbed horizontal velocities due to small angle of attack α are of the order of $\mathrm{O}(\alpha)$ above the wing, become of the order of $\mathrm{O}(\alpha/h)$, in the confined region under the wing. Therefore, the response of the lifting system to the perturbations of the same magnitude is amplified in ground effect as compared to the out-of-ground effect case. It was also shown that in the limit $h \to 0$ a three-dimensional flow problem acquires two-dimensional description in a plane parallel to the ground. Thus, the ground effect theory for $h \to 0$ form an interesting complement to the Prandtl's lifting line theory and Jones's slender body theory in which the flowfield are basically two-dimensional in the transverse and longitudinal planes respectively. The problem of minimization of induced drag for a range of ram wing transportation vehicles, variety of guideway configurations and small relative ground clearances was discussed by Barrows and Widnall [13]. In particular, it was shown that the optimal loading distribution in extreme ground effect is parabolic rather than elliptic as in the infinite fluid case.

Extension of the MAE approach of Widnall and Barrows [15] to a linear and nonlinear flow problem with account of compressibility and unsteadiness was carried out by Rozhdestvensky [18, 19, 20]. The flow field was conditionally subdivided into several domains having different characteristic length scales. Asymptotic expansions derived in these regions were then matched to obtain the uniformly valid solution by additive composition. Therewith, closed form solution were given for a rectangular wing of arbitrary aspect ratio versus Strouhal and Mach number. Analytical expressions were obtained for the lift, moment and induced drag coefficients in heaving and pitching motions with and without account of the leading edge suction force. Systematic analysis of nonlinear unsteady 3-D flow past a lifting system of finite thickness in curved ground effect was summarized in Rozhdestvensky [21]. Angle of pitch, relative thickness and curvature, amplitudes of oscillations and deformations of a wing were assumed to have the order of the characteristic relative clearance h, measured at the trailing edge root section of the wing. In 1991 a similar theory was published by Wang [22] featuring the incompressible flow around a thin lifting surface in curved ground, although no calculated results were presented. Interesting leading order nonlinear formulations of the problem were developed by Tuck for a two-dimensional unsteady [23] and three-dimensional steady [24] incompressible flows. Newman [25] was able to represent the channel flow beneath the lifting surface of small aspect ratio in form of a simple nonlinear solution in a cross flow with appropriate conditions applied at the leading and the trailing edges.

Kida and Miyai [26] applied the MAE approach to account for the lateral curvature of the wing within linearized formulation and $h \to 0$. Later, they extended this approach to the case of a linearized flow past a wing with a jet flap moving in close proximity to the ground, [27].

Efremov *et al.* [28,29, 30] used a variant of the extreme ground effect theory under the name of the *asymptotics of small clearances* to investigate the effects of compressibility of the flow, flexibility and elasticity of the foil, response of a lifting system to unsteady periodic (oscillations) and aperiodic (abrupt change of the angle of attack, influence of step-type vertical gust). In [30] he applied a linearized version of the theory to predict aeroelastic phenomena for some schematized lifting ground-effect flows. To relate elastic displacements of the wing surface with aerodynamic loads he used the equation of unsteady bending of an elastic plate. Lifenko and Rozhdestvensky [31] used the same approach to study aeroelasticity of a lifting surface of finite aspect ratio and rectangular planform in extreme ground effect. To solve the corresponding combined aeroelastic equation they use Bubnov-Galerkin method. Results were presented to describe both static and dynamic stability of an elastic lifting surface in ground effect in a parametric space. It was shown in [30, 31] that speeds of divergence and flutter of the elastic lifting surface in ground effect are less than those out of the ground effect.

A NONLINEAR UNSTEADY FLOW MODEL FOR A LIFTING SURFACE IN EXTREME GROUND EFFECT

Consider a wing of small thickness and curvature performing an unsteady motion above a solid non-planar underlying surface in ideal incompressible fluid. Assume that motion of the wing is a result of a superposition of the main translational motion with variable speed $U(t)$ and small vertical motions due to heave, pitch and deformations of the lifting surface. Introduce a moving coordinate system in which the axes x and z are located upon the unperturbed position of the underlying boundary (the ground). Axis-x is directed forward in the plane of symmetry of the wing, axis-y is directed upward and passes through the trailing edge of the root chord.

All quantities and functions are rendered nondimensional with use of the root chord c_o and a certain characteristic speed U_o. Define relative ground clearance h_o as the ratio of characteristic distance of the trailing edge of the wing from unperturbed position of the ground to the length of the root chord. Introduce functions $y_u(x,z,t), y_l(x,z,t), y_g(x,z,t)$ and $y_w(x,z,t)$, describing respectively instantaneous positions of the upper and lower surfaces of the wing, surfaces of the ground and the wake with respect to the plane $y = 0$. Introduce a small parameter φ to characterize a perturbation, e.g. angle of pitch, curvature, thickness, amplitude of oscillations or deformations of the wing, deformation of the ground surface, *etc.*

In order to develop an asymptotic theory valid in close proximity to the ground, assume that the relative ground clearance is small, that is $h_o << 1$. Assume that at any moment of time the instantaneous distances of the points of the wing, wake and ground surfaces from $y = 0$ are of the same order as h_o, and are changing smoothly in longitudinal and lateral directions. Thus, if $y(x,z,t)$ describes any of these surfaces, then

$$\left(y, \frac{\partial y}{\partial x}, \frac{\partial y}{\partial z}\right) = \mathrm{O}(\varepsilon) = \mathrm{O}(h_o) << 1 \tag{1}$$

It should be noted that in the case of extreme ground effect the adopted assumption $\varepsilon = \mathrm{O}(h_o)$ does not mean that the flow perturbations are necessarily small. It will be shown later that in the extreme ground effect the input of the order $O(\varepsilon)$ can result in the system's response of the order $O(1)$.

The inviscid incompressible flow around a wing in ground effect is governed by three-dimensional Laplace equation and is subject to : flow tangency condition on the surfaces of the wing and the ground; dynamic and kinematic boundary conditions on the surface of the wake (pressure and normal velocity should be continuous across the wake); decay of perturbations at the infinity. The Kutta-Zhukovsky requirement of the pressure continuity should also be specified at the trailing edge, although it is automatically included through the boundary conditions upon the wake surface.

With this in mind we can write the following flow problem formulation with respect to the perturbation velocity potential φ

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}+\frac{\partial^2\varphi}{\partial z^2}=0, \tag{2}$$

$$\frac{\partial\varphi}{\partial y}=\Big[\frac{\partial\varphi}{\partial x}-U(t)\Big]\frac{\partial y_{u,l}}{\partial x}+\frac{\partial\varphi}{\partial z}\frac{\partial y_{u,l}}{\partial x}+\frac{\partial y_{u,l}}{\partial t}$$

$$y=y_{u,l}(x,z,t), \qquad (x,z)\in S; \tag{3}$$

$$\frac{\partial\varphi}{\partial y}=\Big[\frac{\partial\varphi}{\partial x}-U(t)\Big]\frac{\partial y_g}{\partial x}+\frac{\partial\varphi}{\partial z}\frac{\partial y_g}{\partial x}+\frac{\partial y_g}{\partial t}$$

$$y=y_g(x,z,t), \qquad (x,z)\in G; \tag{4}$$

$$p^-=p^+, \qquad (\nabla\varphi\,\vec{n})^-=(\nabla\varphi\,\vec{n})^+, \qquad y=y_w(x,z,t); \tag{5}$$

$$\nabla\varphi\to 0, \qquad x^2+y^2+z^2\to\infty, \tag{6}$$

where S and W are areas of the wing and the wake related to the square of the root chord, G is the ground plane.

According to the technique of matched asymptotics, the flow domain is subdivided into the following regions with different characteristic length scales

- Upper flow region D_u above the wing, it's wake and part of the ground exterior to the projection of the wing and the wake onto unperturbed ground plane
- Channel flow region D_l under the wing and the wake
- Edge flow regions D_e in the vicinity of edges of the lifting surface and the wake.

In each of the regions asymptotic solutions of the problem (1)-(6) are constructed for $h_o\to 0$ in appropriately stretched coordinates. Asymptotic matching and additive composition of these solutions provide a uniformly valid solution for the entire flow domain with asymptotic accuracy of the order of $O(h_o)$.

Omitting details of the analysis leading to the solution of the order of $h_o\to 0$, we shall discuss herein a leading order solution, corresponding to the case of the extreme ground effect. It can be shown that for $h_o\to 0$ the main contribution of the order of $O(1)$ to the aerodynamics of the lifting surface comes from the *channel* flow (lower side of the wing) whereas the upper side and edge regions add quantities of the order of $O(h_o)$. Hencewith, we shall focus attention on derivation of the equations for the channel flow.

In the *channel* flow region D_l, where $\varphi\sim\varphi_l, x=O(1), z=O(1), y=O(h_o)$, introduce the following formulation for the channel flow perturbation potential φ_l

$$\frac{\partial^2\varphi_l}{\partial\bar{y}^2}+h_o^2\Big(\frac{\partial^2\varphi_l}{\partial\bar{x}^2}+\frac{\partial^2\varphi_l}{\partial\bar{z}^2}\Big)=0, \quad (x,y,z)\in D_l; \tag{7}$$

$$\frac{\partial\varphi_l}{\partial\bar{y}}=h_o^2\Big\{\Big[\frac{\partial\varphi_l}{\partial x}-U(t)\Big]\frac{\partial y_l}{\partial x}+\frac{\partial\varphi_l}{\partial z}\frac{\partial y_l}{\partial x}+\frac{\partial y_l}{\partial t}\Big\}, \quad \bar{y}=\bar{y}_l=\frac{y_l}{h_o} \tag{8}$$

$$\frac{\partial\varphi_l}{\partial\bar{y}} = h_o^2\Big\{\Big[\frac{\partial\varphi_l}{\partial x} - U(t)\Big]\frac{\partial y_g}{\partial x} + \frac{\partial\varphi_l}{\partial z}\frac{\partial y_g}{\partial x} + \frac{\partial y_g}{\partial t}\Big\} \quad \bar{y} = \bar{y}_g = \frac{y_g}{h_o}, \tag{9}$$

where φ_u is the perturbation potential in the upper flow region D_u

In the channel flow region both the condition at infinity (decay of perturbations in 3-D flow) and Kutta-Zhukovsky condition at the trailing edge are lost. Influence of these conditions is transferred to the channel flow region D_e by means of matching with asymptotic solutions with the regions D_e and D_u.

Seek φ_l in form of the following asymptotic expansion

$$\varphi_l = \varphi_l^* + h_o^2\varphi_l^{**} = \varphi_{l_1} + \varphi_{l_2} h_o \ln\frac{1}{h_o} + \varphi_{l_3} h_o + h_o^2\varphi_l^{**}, \qquad (\varphi_l^*, \varphi_l^{**}) = \mathrm{O}(1), \tag{10}$$

which can be shown to satisfy the requirement of matching of asymptotic representations of the velocity potentials in the regions D_l, D_u and D_e. Substituting (10) in the equation (7), we come to the following equations for the functions φ_l^* and φ_l^{**}

$$\frac{\partial^2\varphi_l^*}{\partial\bar{y}^2} = 0, \qquad (x, \bar{y}, z) \in D_l; \tag{11}$$

$$\frac{\partial^2\varphi_l^*}{\partial\bar{y}^2} = \frac{\partial^2\varphi_l^*}{\partial x^2} + \frac{\partial^2\varphi_l^*}{\partial z^2}, \qquad (x, \bar{y}, z) \in D_l; \tag{12}$$

Then, using the same asymptotic expansion (10) in the flow tangency conditions on the lower surface of the wing (9), we come to the following set of equations

– on the lower surface of the wing

$$\frac{\partial\varphi_l^*}{\partial\bar{y}} = 0, \qquad \bar{y} = \bar{y}_l(x, z, t); \tag{13}$$

$$\frac{\partial\varphi_l^* *}{\partial\bar{y}} = \Big[\frac{\partial\varphi_l^*}{\partial x} - U(t)\Big]\frac{\partial y_l}{\partial x} + \frac{\partial\varphi_l^*}{\partial z}\frac{\partial y_l}{\partial x} + \frac{\partial y_l}{\partial t}, \quad \bar{y} = \bar{y}_l(x, z). \tag{14}$$

– on the ground

$$\frac{\partial\varphi_l^*}{\partial\bar{y}} = 0, \qquad \bar{y} = \bar{y}_g(x, z, t); \tag{15}$$

$$\frac{\partial\varphi_l^* *}{\partial\bar{y}} = \Big[\frac{\partial\varphi_g^*}{\partial x} - U(t)\Big]\frac{\partial y_g}{\partial x} + \frac{\partial\varphi_l^*}{\partial z}\frac{\partial y_g}{\partial z} + \frac{\partial y_g}{\partial t}, \quad \bar{y} = \bar{y}_g(x, z, t); \tag{16}$$

– Integrating (11) two times with respect to $\bar{y}$ and accounting for (13) and (14) we come to important conclusion : *with asymptotic error of the order of* $O(h_o)$ description of the channel flow is two-dimensional in the plane parallel to unperturbed position of the ground surface, i.e. to the plane $y = 0$

$$\varphi_l^* = \varphi_l^*(x, z, t). \tag{17}$$

Integrating (12) one time with respect to $\bar{y}$, we have

$$\frac{\partial\varphi_l^{**}}{\partial\bar{y}} = \Big(\frac{\partial^2\varphi_l^*}{\partial x^2} + \frac{\partial^2\varphi_l^*}{\partial z^2}\Big)\bar{y} + f^{**}(x, z), \tag{18}$$

where $f^{**}(x, z)$ is an unknown function. Taking into account equations (14) and (16) we obtain

$$\Big(\frac{\partial^2\varphi_l^*}{\partial x^2} + \frac{\partial^2\varphi_l^*}{\partial z^2}\Big)\bar{y}_l + f^{**}(x, z) = \Big[\frac{\partial\varphi_l^*}{\partial x} - U(t)\Big]\frac{\partial\bar{y}_l}{\partial x} + \frac{\partial\varphi_l^*}{\partial z}\frac{\partial\bar{y}_l}{\partial z} + \frac{\partial\bar{y}_l}{\partial t},$$

$$\bar{y} = \bar{y}_l(x, z, t) \tag{19}$$

$$\Big(\frac{\partial^2\varphi_l^*}{\partial x^2} + \frac{\partial^2\varphi_l^*}{\partial z^2}\Big)\bar{y}_g + f^{**}(x, z) = \Big[\frac{\partial\varphi_l^*}{\partial x} - U(t)\Big]\frac{\partial\bar{y}_g}{\partial x} + \frac{\partial\varphi_l^*}{\partial z}\frac{\partial\bar{y}_g}{\partial z} + \frac{\partial\bar{y}_g}{\partial t},$$

$$\bar{y} = \bar{y}_g(x, z, t) \tag{20}$$

Substracting (20) from (19) we come to the following channel flow equation

$$\frac{\partial}{\partial x}\Big(\bar{h}^* \frac{\partial \varphi_l^*}{\partial x}\Big) + \frac{\partial}{\partial z}\Big(\bar{h}^* \frac{\partial \varphi_l^*}{\partial z}\Big) = U(t)\frac{\partial \bar{h}^*}{\partial x} - \frac{\partial \bar{h}^*}{\partial t}, \qquad (x, z) \in S, \tag{21}$$

where $\bar{h}^*(x, z, t) = h^*(x, z, t)/h_o = \bar{y}_l(x, y, t) - \bar{y}_g(x, y, t)$ instantaneous distribution of the gap between the wing and the ground.

Thus, for very small clearances the flow field under the wing has a two-dimensional description, and its perturbation velocity potential $\varphi_l \sim \varphi_l^*$ satisfies quasi-harmonic equation (21) in a two-dimensional domain S bounded by the wing planform contour.

Boundary conditions for φ_l^* at the leading edge l_1 and trailing edge l_2 of the lifting surface should be obtained by means of matching.

From physical viewpoint the equation (21) can be interpreted as that of mass conservation in the highly constrained channel flow region with known distributed mass addition due to the tangency conditions on the lower side of the wing and part of the ground, situated under the wing.

For the channel flow under the wake the same procedures can be utilized to reduce the induced downwash $\alpha = \mathrm{O}(h_o)$ in the wake

$$\alpha_w = h_o \bar{\alpha}_w = h_o\Big(\alpha_{w_1} + \alpha_{w_2} h_o \ln \frac{1}{h_o} + \alpha_{w_3} h_o\Big) \tag{22}$$

to the wake channel flow potential φ_l^* and the corresponding instantaneous gap distribution $h_w^*(x, z, t) = y_l(x, y, t) - y_g(x, y, t)$ by means of the following equation

$$\alpha_w = h_o \bar{\alpha}_w = h_o\Big[\frac{\partial}{\partial x}\Big(\bar{h}_w^* \frac{\partial \varphi_l^*}{\partial x}\Big) + \frac{\partial}{\partial z}\Big(\bar{h}_w^* \frac{\partial \varphi_l^*}{\partial z}\Big)\Big], \qquad (x, z) \in W, \tag{23}$$

where $\bar{h}_w^* = (y_w - y_g)/h_o$.

To the lowest order (extreme ground effect) the flow problem for a wing in the ground effect takes simple form even for the case of curved ground

– Asymptotic orders of the upper and lower surface velocity potentials

$$\varphi^- = \varphi_l^* + \mathrm{O}(h_o \ln \frac{1}{h_o}), \qquad \varphi^+ = \mathrm{O}(h). \tag{24}$$

– Channel flow equation for extreme ground effect

$$\frac{\partial}{\partial x}\Big(\bar{h}^* \frac{\partial \varphi_{l_1}}{\partial x}\Big) + \frac{\partial}{\partial z}\Big(\bar{h}^* \frac{\partial \varphi_{l_1}}{\partial z}\Big) = U(t)\frac{\partial \bar{h}^*}{\partial x} - \frac{\partial \bar{h}^*}{\partial t}. \tag{25}$$

– Boundary condition at the leading (side) edge l_1

$$\varphi_{l_1} = 0, \qquad (x, z) \in l_1. \tag{26}$$

– Boundary condition at the trailing edge l_2

$$2\Big[\frac{\partial \varphi_{l_2}}{\partial x} U(t) - \frac{\partial \varphi_{l_1}}{\partial t}\Big] - \Big(\frac{\partial \varphi_{l_1}}{\partial x}\Big)^2 - \Big(\frac{\partial \varphi_{l_1}}{\partial z}\Big)^2 = 0, \qquad (x, z) \in l_2. \tag{27}$$

Boundary conditions (26), (27) can be obtained by means of asymptotic matching of the upper flow with the channel flow through edge flow regions. From physical viewpoint they results from continuity of the velocity potential at the leading edge (no vorticity in the incoming flow) and pressure at the trailing edge (Kutta-Zhukovsky condition) with account of the order of contributions of the upper flow and the channel flow. Formulation of the conditions at side edges is optional and depends on the adopted model of the local flow.

ONE-DIMENSIONAL NONLINEAR FLOW MODEL FOR A WING WITH ENDPLATES IN EXTREME GROUND EFFECT

In what follows we shall confine ourselves to consideration of a flow past a rectangular wing of aspect ratio λ with endplates in close proximity to the ground. In this case, as demonstrated by numerous experiments and theoretical calculations, the cruise performance of a properly designed vehicle improves quite considerably. This enhancement of performance is due to the fact that for small ground clearances the flow leakage through the gaps under tips of the endplates is hampered, and, by consequence, the effective aspect ratio of the wing becomes large , no matter how small may be its geometric aspect ratio. As shown in Rozhdestvensky [20], the corresponding mathematical model of the flow can be reduced to that in one dimension for both steady and unsteady cases. Herein, the analysis is restricted to the steady case and the derivation is carried out in a somewhat more general fashion. Consider a steady flow past a rectangular lifting surface with endplates in close proximity to the flat ground. As seen from the previous section, when h tends to zero, the leading order problem for the potential of steady relative motion of the fluid can be reduced to a quasi-harmonic equation

$$\frac{\partial}{\partial x}\Big(\bar{h}^*\frac{\partial\varphi}{\partial x}\Big)+\frac{\partial}{\partial z}\Big(\bar{h}^*\frac{\partial\varphi}{\partial z}\Big)=0 \tag{28}$$

Introduce spanwise averaging of the local ground clearance and longitudinal velocity of the fluid in the "channel" between the wing and ground

$$\hat{h}(x)=\frac{1}{\lambda}\int_{-\lambda/2}^{\lambda/2}\bar{h}^*(x,z)\,\mathrm{d}z,\quad \hat{v}(x)=\frac{1}{\lambda\hat{h}^*(x,z)}\int_{-\lambda/2}^{\lambda/2}\bar{h}^*(x,z)\frac{\partial\varphi(x,z)}{\partial x}\,\mathrm{d}z. \tag{29}$$

Averaging the equations (29) along the span, we have

$$\frac{d}{\mathrm{d}x}[\hat{h}(x)\hat{v}(x)]+\frac{1}{\lambda h}\Big[h^*(x,\frac{\lambda}{2})\frac{\partial\varphi}{\partial z}(x,\frac{\lambda}{2})-h^*(x,-\frac{\lambda}{2})\frac{\partial\varphi}{\partial z}(x,-\frac{\lambda}{2})\Big]=0. \tag{30}$$

Relate the velocities v_{epr} and v_{epl} of *leakage* from under the right-hand side and the left-hand side endplates to the spanwise averaged longitudinal velocity $\hat{v}(x)$, assuming therewith that the perturbed pressure outside of the endplates is equal to zero. Let $\delta_{epr}(x)$ and $\delta_{epl}(x)$ be *effective gaps* under the endplates. Then the velocities of leakage form under the wing become

$$v_{epr}(x)=\frac{h^*(x,\lambda/2)}{\delta_{epr}(x)}\frac{\partial\varphi}{\partial z}(x,\frac{\lambda}{2}),\qquad v_{epl}(x)=-\frac{h^*(x,-\lambda/2)}{\delta_{epl}(x)}\frac{\partial\varphi}{\partial z}(x,-\frac{\lambda}{2}) \tag{31}$$

and the adopted dynamic condition just outside the endplate takes the form

$$\hat{p}(x)=1-v_{ep}^2(x)-\Big[\frac{\partial\varphi(x)}{\partial x}\Big]^2=0, \tag{32}$$

wherefrom

$$\frac{\partial\varphi}{\partial z}(x,\frac{\lambda}{2})=\frac{\delta_{epr}(x)}{h^*(x,\lambda/2)}\mathrm{sign}\,\hat{p}(x)\sqrt{|\hat{p}(x)|},$$

$$\frac{\partial\varphi}{\partial z}(x,-\frac{\lambda}{2})=-\frac{\delta_{epl}(x)}{h^*(z,-\lambda/2)}\mathrm{sign}\,\hat{p}(x)\sqrt{|\hat{p}(x)|}, \tag{33}$$

where $\hat{p}(x)=1-\hat{v}^2(x)$ is the spanwise averaged pressure coefficient under the wing.

Now, with account of (33) the equation (30) takes the form

$$\frac{\mathrm{d}}{\mathrm{d}x}\Big[\hat{h}(x)\hat{v}(x)\Big]+\frac{\delta_{epr}+\delta_{epl}}{\lambda h}\mathrm{sign}\,\hat{p}(x)\sqrt{|\hat{p}(x)|}=0. \tag{34}$$

Note that the sign-function in front of the square root in equation (34) accounts for the direction of leakage at a station x chordwise. The outward leakage takes place in the case of positive pressure under the wing, and the inward flow leakage occurs in the case of suction under the wing. Equation (34) is an ordinary differential equation of the first order with respect to the function $\hat{v}(x)$. As a matter of fact, this equation is an expression of mass conservation in the "channel" relating flow rates in longitudinal and lateral directions. The boundary condition at the trailing edge, accounting for the Kutta-Zhukovsky requirement can be written as

$$\hat{v}(0) = -\frac{\delta_f}{h} = \bar{\delta}_f. \tag{35}$$

Parameter $\bar{\delta}_f = \delta_f/h$ is an effective gap under the rear flap (related to ground clearance h). The one-dimensional flow model, derived above, describes aerodynamics of a *flying wing configuration* with endplates in steady extreme ground effect. It should be underlined that the mathematical model under consideration incorporates inherent nonlinearity of the ground effect phenomena as regards the geometry and kinematics of the lifting system. This property of the model is provided due to the assumption that all perturbation parameters (incidence, curvature, *etc.*) have the same order of magnitude as ground clearance. Considerable simplification of the flow model for a wing with endplates for $h \to 0$ has a common sense basis. When the endplate tip clearances are very small, the channel flow, which is already "squeezed" vertically due to vanishing ground clearance, tends to become almost one-dimensional. It should be kept in mind that the equation (34) describes the main contribution to aerodynamics of the lifting system, namely, that of the channel flow. For $h \to 0$ this contribution is of the order of O(1). Contribution of the upper (suction) side of the wing can be shown to be of the order of h, that is smaller. The coefficients of lift C_y and longitudinal moment m_z around the trailing edge can be determined by means of the formulae

$$C_y = \int_0^1 \hat{p}(x)\,\mathrm{d}x, \qquad m_z = \int_0^1 x\hat{p}(x)\,\mathrm{d}x. \tag{36}$$

Induced drag coefficient is defined herein as the difference between the pressure drag in inviscid flow and suction force, acting upon the leading edge. The induced drag coefficient C_{x_i} is thus written as

$$C_{x_i} = C_{x_p} + C_s \tag{37}$$

where C_{x_p} is an ideal pressure drag due to the action of the aerodynamic forces normal to the wing surface, C_s is a suction force coefficient. For a wing with a short rear flap this coefficient can be obtained with help of the formula

$$C_{x_p} = -h\Big[\int_0^1 \hat{p}(x)\frac{\partial y_w}{\partial x}\,\mathrm{d}x + \Big(1 - \frac{\delta_f}{h}\Big)^2\Big]. \tag{38}$$

The expression for the suction force coefficient, acting upon the leading edge of a wing with endplates in extreme ground effect can be derived in the form

$$C_s = h\hat{h}(1)[1 + \hat{v}(1)]^2, \tag{39}$$

where $h\hat{h}(1)$ and $\hat{v}(1)$ are span averaged relative ground clearance and the *channel flow* velocity at the leading edge.

Mathematical models of extreme ground effect reveal certain *similarity criteria* which are important for design. We shall now discuss some of these criteria in the case of zero angle of heel and uniform chordwise distribution of gap under the tips of the endplates. With $\delta_{epr} = \delta_{epl} = \delta_{ep} = \delta^o_{ep} = constant$ it is convenient to re-write the equation (34) in the form

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\hat{h}(x)\hat{v}(x)\right] + G\,\mathrm{sign}\,\sqrt{|\hat{p}(x)|} = 0. \tag{40}$$

It can be seen through comparison of equations (34) and (40) that for the case of constant gap under the tips of the endplates the solution will depend on a similarity parameter

$$G = \frac{2\delta^o_{ep}}{\lambda h} \tag{41}$$

rather than separately on the gap under the endplates δ^o_{ep}, aspect ratio λ and relative ground clearance h. Parameter G can be called a *generalized gap parameter*. Besides, the solution will depend on the ratios which have the form ε/h, where ε is a parameter, characterizing geometry or kinematics of the lifting system (e.g. angle of pitch, maximum curvature of the wing, effective gap under the rear flap, *etc.*). Now, the solution of the flow problem for the case of constant gap under the endplates can be defined in terms of a set of similarity criteria. This circumstance simplifies representation and processing of theoretical and experimental data, and, consequently, becomes instrumental for conceptual and preliminary design of the craft, utilizing the ground effect. In some practical cases the solution of equation (41) with condition (39) can be obtained in analytical form. In what follows we shall refer to some examples of closed form solution. Consider first a flat plate at zero incidence at a constant clearance and a short rear flap[1]. In this case the following expression can easily obtained for the lift, moment and induced drag coefficient

$$C_y = \frac{1}{2} + \frac{1}{2G}\cos(G + 2\arcsin\bar{\delta}_f)\sin G, \tag{42}$$

$$m_z = \frac{1}{4}\Big\{1 + \frac{1}{G}[\sin(2G + 2\arcsin\bar{\delta}_f)] +$$

$$\frac{1}{2G}[\cos(2G + 2\arcsin\bar{\delta}_f) - \cos(2\arcsin\bar{\delta}_f)]\Big\} \tag{43}$$

$$C_{x_i} = \frac{C_{x_i}}{h} = [1 - \sin(G + \arcsin\bar{\delta}_f)]^2 - (1 - \bar{\delta})^2 \tag{44}$$

As seen from the above formulae, aerodynamic characteristics of the lifting system depend only on two parameters, namely G and $\bar{\delta}_f$ whereas the total number of parameters is equal to 4, including relative clearance h, aspect ratio λ, effective gap under the endplates δ^o_{ep} and effective gap under the rear flap $\bar{\delta}_f$. In this case, use of similarity criteria reduces the number of essential parameters of the problem two-fold. Generally, for a constant endplate gap, the number of parameters, characterizing the problem will be $n - 2$, where n is the initial number of parameters. Another integrable case is that of a flat plate of pitch angle θ with endplates at constant clearance and a short rear flap. In this case the ground clearance function $\hat{h}(x) = 1 + \bar{\theta}x, \bar{\theta} = \theta/h$, and the equation can be represented in a separable form

$$\frac{\mathrm{d}\hat{v}}{\hat{v} + G_\theta\sqrt{1 - \hat{v}^2}} = -\frac{\bar{\theta}\mathrm{d}x}{1 + \bar{\theta}x}, \qquad G_\theta = \frac{2\delta^o_{ep}}{\lambda\theta}, \tag{45}$$

where G_θ is an alternative similarity parameter similar to the parameter H, introduced by Gallington [16]. Some calculations for this example can be found in Rozhdestvensky [20, 21].

The one-dimensional mathematical flow model under discussion contains as a concrete case the *slit orifice* theory derived by Gallington *et al*, [16]. These authors considered a rectangular wing in

[1] Here the term “short”implies that the chord of the flap is of the order of h

the form of a flat plate, assumed that the pressure remains constant along the chord and applied elementary continuity considerations. The *slit-orifice theory* is very simple but agrees well with experimental data.

To retrieve the *slit-orifice* theory from the general flow model, discussed herein, suppose that there exists a solution of (40) for which the span-averaged velocity $\hat{v}$ (pressure $\hat{p}$) is constant along the chord, i.e. $\hat{v}(x) = \hat{v} = const$. Then, for a flat plate with the endplates' gap constant chordwise and at a pitch angle θ the equation (40) can be written as

$$\hat{v} + G_\theta\sqrt{1-\hat{v}^2} = 0, \qquad G_\theta = \frac{2\delta^o_{ep}}{\lambda\theta}. \tag{46}$$

The above equation is easily solved with respect to $\hat{v}$ so that

$$\hat{v} = -\frac{G_\theta}{\sqrt{1+G_\theta^2}}, \qquad \hat{p} = \frac{1}{1+G_\theta^2}. \tag{47}$$

Equation (47) defines the pressure coefficient in terms of parameter G_θ, i.e. in terms of the combination of : the gap under endplates, aspect ratio of the wing and adjusted pitch angle. On the other hand, it gives a simple tool to design a ram wing for a given pressure in the *channel*. At the same time, the assumption of constant $\hat{v}$ together with prescribed boundary condition (35) necessitates the following relationships

$$\hat{p} = 1 - \bar{\delta}^2 = 1 - \left(\frac{\delta}{h}\right)^2 \tag{48}$$

$$\bar{\delta}_f = \frac{\delta_f}{h} = \frac{G_\theta}{1+G_\theta^2} = \sqrt{1-\hat{p}} \tag{49}$$

which indicate that the rear flap setting should be "tuned up" with the parameter G_θ (or prescribed pressure in the dynamic air cushion).

It shows in particular that the increase of design flight clearance for the same magnitude of loading should be followed by opening of the gap. Some results of calculations of the lift and drag coefficients are presented in Fig. 1 and 2 in comparison with experimental results and *slit-orifice* theory of [16] for the case of rectangular wing of aspect ratio $\lambda = 0.5$ with endplates, short trailing edge flap and adjusted pitch angle $\theta = 0.06$ (radians)

Note that the induced drag coefficients are determined differently in *slit-orifice* theory and the present approach. In the former case the drag law was obtained through application of conservation of momentum, whereas in the latter case the induced drag was obtained by a straightforward integration of pressures upon the wing incorporating contributions of the suction force and the pressure drag of the trailing edge gap. In the calculations the upper side lift contribution and the profile (viscous) drag component were accounted for on the basis of the experimental data of [16]. Fig. 1 gives comparison of lift-to-drag ratio of the aforementioned wing as well as for another rectangular wing model tested in [16] which had aspect ratio $\lambda = 2/3$. Divergence between the theory and the test data on lift-to-drag ratio in the second case is due to separation of the flow as reported in [16].

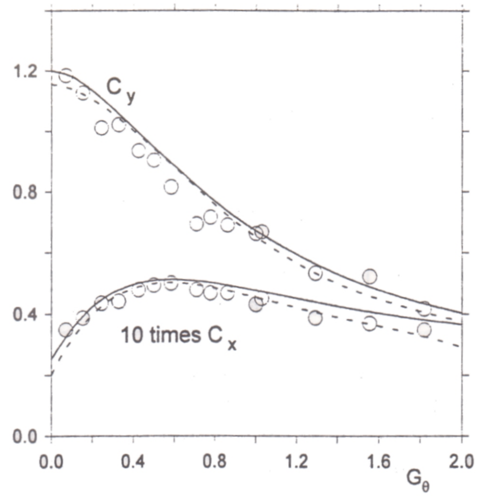

Figure 1 : The lift and drag coefficients of a rectangular wing with endplates versus parameter $G_\theta = 2\delta^o_{\lambda\theta}$: theory and experiment ($\lambda = 0.5$; circles : experiment [16] ; solid lines : present theory ; dashed lines : *slit-orifice* theory)

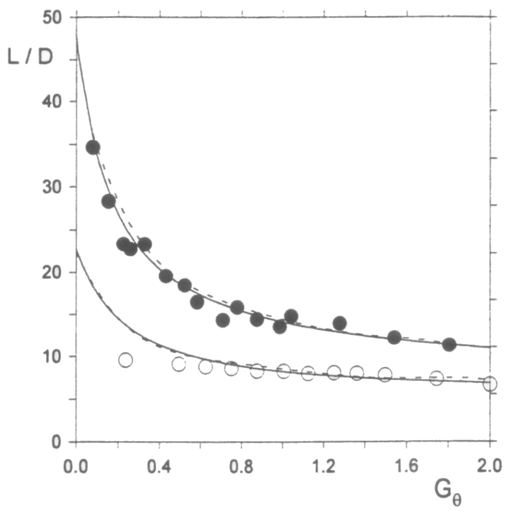

Figure 2 : The lift-to-drag ratio of rectangular wings with endplates : theory and experiment (black circles : experiment for $\lambda = 2/3$, [16] ; empty circles for $\lambda = 0.5$, [16] ; dashed lines : *slit-orifice* theory)

Finally, let's turn over to the (simplest) *case of zero endplate tip clearance.* Note that due to the structure of the basic equation (40) this case corresponds to $G = 0$ which can be achieved by either putting the aspect ratio of the wing tend to infinity ($\lambda \to \infty$). Then, the equation (34) yields

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\bar{h}(x)\hat{v}(x)\right] = 0, \qquad \hat{v}(0) = -\hat{v}(0) = -\bar{\delta}_f \tag{50}$$

and has elementary solution

$$\hat{v}(x) = \frac{\bar{\delta}_f}{\bar{h}(x)}, \qquad \hat{p} = 1 - \left[\frac{\bar{\delta}_f}{\bar{h}(x)}\right]^2. \tag{51}$$

Aerodynamic coefficients are calculated by means of the formulae

$$C_y = 1 - \bar{\delta}_f^2 \int_0^1 \frac{\mathrm{d}}{\bar{h}^2(x)}, \quad m_z = \frac{1}{2} - \bar{\delta}_f^2 \int_0^1 \frac{x\,\mathrm{d}x}{\bar{h}^2(x)}, \quad x_p = \frac{m_z}{C_y}. \tag{52}$$

In what follows the above theory is applied to qualitative analysis of static stability stability of longitudinal motion of a single wing in extreme ground effect.

AN ESTIMATE OF STATIC STABILITY OF A SINGLE WING IN EXTREME GROUND EFFECT

As shown by Irodov [32], Staufenbiel [33] and Zhukov [34] the longitudinal static stability of motion of a wing-in-ground effect vehicle depends on reciprocal location of the aerodynamic centers in height and in pitch, as well as upon location of the center of gravity. Define positions of aerodynamic centers in height in height and pitch correspondingly as

$$x^h = \frac{m_z^h}{C_y^h}, \qquad x^\theta = \frac{m_z^\theta}{C_y^\theta} \tag{53}$$

where the superscripts h and θ are ascribed correspondingly to the derivatives of lift and moment coefficients with respect to ground clearance and angle of pitch. Analyzing linearized equations of longitudinal motion of wing-in-ground effect vehicles, Irodov has shown that the static stability is ensured if the aerodynamic center in height is located upstream of the aerodynamic center in pitch, so that for x-axis directed upstream the corresponding static stability criterion can be written as

$$x^h - x^\theta > 0 \tag{54}$$

As a matter of fact the Irodov's criterion deals with pitch stability, implicitly assuming that the height stability is ensured, i.e. $C_y^h < 0$. Let us designate the above difference in location of the centers as $SSM = x^h - x^\theta$ and, when SSM is positive, refer to it as a *static stability margin.* Suppose that both derivatives (in height and in pitch) and the position of the corresponding centers are defined with respect to the center of gravity which can be viewed as a pivotal point. Introducing abscissa of the latter x_g and noting that changing reference point does not affect differentiation with respect to h whereas

$$\frac{\partial}{\partial \theta} \to \frac{\partial}{\partial \theta} - x_g \frac{\partial}{\partial h}, \tag{55}$$

we arrive at the following formulae for the new positions of centers in height and in pitch in the case when the reference point coincides with the center of gravity expressed through corresponding parameters referred to the trailing edge

$$x_g^h = x^h, \qquad x_g^\theta = \left(\frac{m_z^\theta}{C_y^\theta}\right)_g = \frac{m_z^\theta - x_g m_z^h}{C_y^\theta - x_g C_y^h} = \frac{\mathcal{K} x^\theta - x_g x^h}{\mathcal{K} - x_g}, \tag{56}$$

where $\mathcal{K} = C_y^\theta / C_y^h$. For a foil possessing height stability and $C_y^\theta / C_y^h > 0$ the factor $\mathcal{K}$ is negative. It may be practical to evaluate variation of the static stability margin as a function of the position of the center of gravity (pivotal point). Simple calculations lead to the following equation

$$x_g^h - x_g^\theta = x^h - \frac{\mathcal{K} x_\theta - x_g x^h}{\mathcal{K} - x_g} = \frac{\mathcal{K}}{\mathcal{K} - x_g}(x^h - x^\theta) \quad \to SSM_g = \frac{\mathcal{K}}{\mathcal{K} - x_g} SSM. \tag{57}$$

It can be seen from previous equations that if a wing is found to be statically stable with reference to the trailing edge, its static stability is ensured for any other upstream position of the reference point (center of gravity), see [32]. Simultaneously, the equation (56) shows that in the case when the center of gravity is shifted upstream from the trailing edge, the center in pitch moves in the same direction whereas the center in height retains its position. Consequently, the static stability margin diminishes. In what follows a qualitative analysis is carried out of static longitudinal stability of simple aerodynamic configurations with use of mathematical models in extreme ground effect.

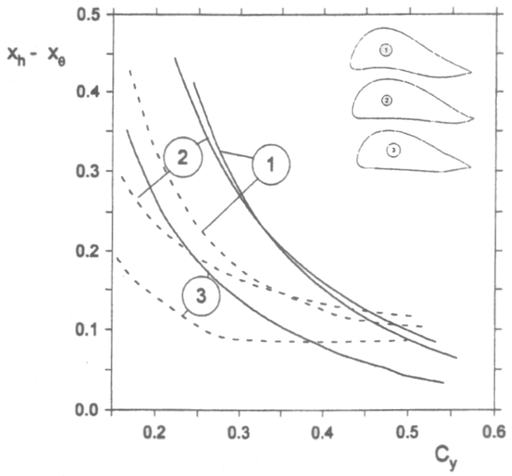

Figure 3 : Static stability margins for foils ($\lambda = \infty, h = 0.1$ solid lines) and rectangular wings with endplates ($\lambda = 0.625, h = 0.1, \delta^o_{ep} = 0.025$). The numbers correspond to 1 : *sine*-foil section ; 2 : *stab*-foil section ; 3 : *delta*-foil section

Consider a simple example of a single foil at full flap opening. As follows from (52) for $\bar{\delta}_f = 1$ the lift and moment coefficients of a single foil in extreme ground effect are given by the formulae

$$C_y = 1 - \int_0^1 \frac{\mathrm{d}x}{\bar{h}^{*2}(x)}, \qquad m_z = \frac{1}{2} - \int_0^1 \frac{x\mathrm{d}x}{\bar{h}^{*2}(x)} \tag{58}$$

where h is a relative ground clearance defined at the trailing edge. Writing $\bar{h}^*(x) = 1 + bar\theta x + \bar{\varepsilon} f(x)$ (where $\bar{\theta} = \theta/h, \bar{\varepsilon} = \varepsilon/h$, $\varepsilon = \mathrm{O}(h)$ is a small parameter characterizing curvature of the lower side of the foil), we can perform differentiation of (58) with respect to h and θ to obtain

$$hC_y^\theta = 2\int_0^1 \frac{x\mathrm{d}x}{\bar{h}^{*3}(x)}, \qquad hm_z^\theta = 2\int_0^1 \frac{x^2\mathrm{d}x}{\bar{h}^{*3}(x)}, \tag{59}$$

$$hC_y^h = 2\int_0^1 \frac{1 - \bar{h}^*(x)}{\bar{h}^{*3}(x)}\,\mathrm{d}x, \qquad hm_z^h = 2\int_0^1 \frac{x[1 - \bar{h}^*(x)]}{\bar{h}^{*3}(x)}\,\mathrm{d}x. \tag{60}$$

It follows from (58) and (59) that the quantities $hC_y^h, hC_y^\theta, hm_z^h, hm_z^\theta$ depend upon $\bar{\theta}$ and $\bar{\varepsilon}$ rather than upon θ, h and ε.In accordance with the initially assumed order relationships between the small parameters the above quantities are of the order of O(1).

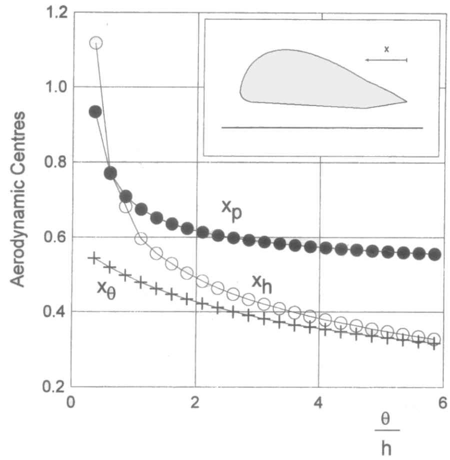

Figure 4 : Positions of aerodynamic centers versus the ratio θ/h for a *delta* foil, $\varepsilon/h = 0.2, x_d = 0.2$

We can calculate position of the centers either using (53) and (60), or alternatively as

$$x^h = \frac{m_z^{\bar\theta}\bar\theta + m_z^{\bar\varepsilon}\bar\varepsilon}{mC_y^{\bar\theta}\bar\theta + C_y^{\bar\varepsilon}\bar\varepsilon}, \qquad x^\theta = \frac{m_z^{\bar\theta}}{C_y^{\bar\theta}}, \tag{61}$$

where

$$C_y^{\bar\theta} = -2\int_0^1 \frac{x\,\mathrm{d}x}{\bar h^{*3}}, \qquad m_z^{\bar\theta} = -2\int_0^1 \frac{x^2\,\mathrm{d}x}{\bar h^{*3}} \tag{62}$$

$$C_y^{\bar\varepsilon} = -2\int_0^1 \frac{f(x)\,\mathrm{d}x}{\bar h^{*3}}, \qquad m_z^{\bar\varepsilon} = -2\int_0^1 \frac{xf(x)\,\mathrm{d}x}{\bar h^{*3}}. \tag{63}$$

It can be concluded from (61) that for a flat plate ($\bar\varepsilon = 0$) the abscissas of the centers in height and pitch coincide. Hence *flat foil in extreme ground effect is not statically stable.* However, static stability of a single foil can be achieved introducing a nonplanar (curved or/and polygonal) lower surface. It follows from the extreme ground effect theory that the stability margin of a curved foil depend upon the ratio of the curvature to the ground clearance rather than upon the curvature. It means than to ensure the same reserve of stability for smaller ground clearances one has to utilize proportionally smaller curvatures. One of the known recipes to improve stability of a single foil is *S-shaping*, see Staufenbiel [33], Gadetski [35], *etc.* A simple representative of such a family is a foil with sinusoidal lower side, described by the equation $-\varepsilon\sin(2\pi x), x \in [0, 1]$, where ε is the amplitude of the sine function related to the chord of the foil. It turns out that other forms of the lower surface can be proposed which also lead to enhancement of stability of longitudinal motion.

In Fig. 3 comparison is presented of the behaviour of the $SSM = x^h - x^\theta$ versus design lift coeffcient for the above mentionned sine foil, a special *stab*-foil with the equation of the lower side $15\varepsilon\, x(1-x)^5$ and a *delta*-foil having the lower side composed of two flat segments joined in a vertex located at 25% of the chord from the trailing edge and $\varepsilon = 0.2h = 0.02$. Plotted in the same Figure are some calculated results for the case of rectangular wings of the finite aspect ratio $\lambda = 0.625$ for the same foil sections, ground clearance and relative curvature. To better demonstrate the form of the foils under discussion their vertical dimension is multiplied by 4. Figures 4 and 5 shows influence of adjusted pitch angle and position of the vertex of the *delta*-foil section.

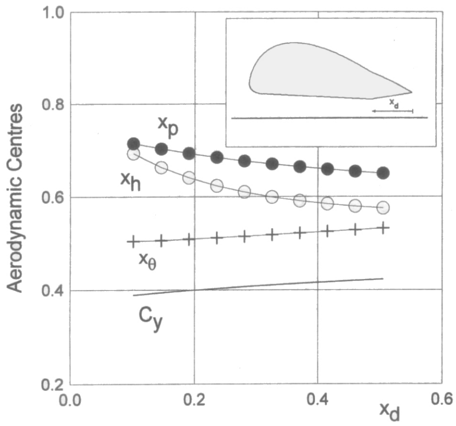

Figure 5 : Influence of position of the vertex of the *delta* foil upon the lift coefficient and the position of the aerodynamic centers : pressure (black circles) ; height (empty circles) ; and pitch (crosses), $\theta/h = 0.1$

CONCLUSIONS

Mathematics of extreme ground effect reveals some peculiarities of aerodynamics of lifting systems in this practically interesting regime of operation, [36]

- The dominating contribution to aerodynamics of a lifting system in extreme ground effect is due to the channel flow under the main wing. Consequently, both aerodynamic efficiency and longitudinal stability of the vehicle largely depend on the geometry of the lower side of the main wing and part of the ground located directly under the main wing.
- At small relative clearances the effective aspect ratio of the wing is a function of three factors : geometric aspect ratio, ground clearance and gaps under the endplates. Thus, design solutions can become remarkably diversified as compared to the out-of-ground-effect case.
- Extreme ground effect is a highly nonlinear phenomenon. In this connection, superposition of different effects, as adopted in aviation, becomes senseless. For example, the effects of thickness and curvature cannot be, strictly speaking, either studied separately or “summed”. Combined influence of thickness and curvature is mainly defined in this case by a resulting form of the lower side of the wing. The nonlinearity may also result in nonzero component of the time-averaged forces which act upon the main wing of the vehicle, performing unsteady motion in close proximity to the ground. The said non-zero time-averaged lift contributions can be directed either toward the ground or from the ground. For example, motion of the main lifting wing with flat lower surface over a wavy underlying surface at an angle of attack gives rise to a non-zero time-averaged lift increment.
- Influence of compressibility is more enhanced than out-of-ground effect. In particular, for a subsonic flow past wings of moderate and large aspect ratios in vicinity of the ground, an augmentation in Mach number results in a larger increment of lift than that in unbounded fluid. To understand how important it may be to account for the compressibility effects when designing a lifting system for ekranoplan, note that the cruising speed of KM constituted 40% of the speed of sound at sea level. At larger speeds the influence of compressibility can become still more dramatic due to possible formation of shock waves

– In close proximity to the ground the influence of the aspect ratio and unsteadiness of the flow upon the main wing is mainly determined by the free vorticity within the wing's planform. The corresponding effects differ from those in unbounded fluid. For example, for $h \to 0$ the dependence of the lift coefficient upon the aspect ratio is quadratic rather than linear as predicted by Jones theory. Unsteady aerodynamic characteristics in extreme ground effect vary more slowly versus Strouhal number than thoes in unbounded fluid case.
– Similarly to the airplanes, one of the reserves of augmentation of lift-to-drag ratio is connected to the realization of the suction force at the leading edge of the main wing. As follows from calculations, for wings of large and moderate aspect ratios, realization of the suction force brings about more noticeable relative increase of the maximum aerodynamic fineness than in the out-of-ground-effect case. Hencewith, the profiling of the leading edges of the lifting system designed to operate in ground effect should be done most thoroughly, and all available boundary layer control should be applied to avoid separation. Note in this connection that when the wing approaches the ground the flow separation from the leading edge becomes more probable.
– Optimal design solutions for $h \to 0$ differ qualitatively from the corresponding unbounded fluid case. Whereas in the latter case the optimal loading distribution is known to be elliptic, for a wing in extreme ground effect the optimal loading becomes parabolic spanwise.
– The bigger the size of ekranoplan, the more attractive the vehicle becomes from viewpoint of fuel consumption and seaworthiness. At the same time, increase of the vehicle's dimensions entails augmentation of aeroelastic effects. Both theoretical and practical data shows that for smaller ground clearance the magnitudes of speeds of divergence and flutter decrease.

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DISCUSSION

Graham Taylor (GT), Independent Consultant

Pr. Rozhdestvensky, I felt it very interesting but I wanted to know if you could just go back for me a non scientist over the issue of G[2] ?

Kirill V. Rozhdestvensky (KVR), Saint Petersburg State Marine Technical University

The physics are that you have the same effect by increasing the aspect ratio, by decreasing the clearance under the tips of the endplates or by decreasing the relative ground clearance. So you can use all these effects to have the same result : high L/D but depending on your specifications, you can use one or another parametre.

Chairman Grégoire Casalis (GC), SUPAERO-ONERA

About viscous effects, do you have any idea of them and do you know the size of the boundary layer ?

KVR

Extreme ground effect does not mean capillary small distances. It only means that the ratio of the distance to the ground to the chord length is relatively small. Usually the boundary layer is still very small with respect to the height. But of course you can use displacement thickness correction which gives another line for the lower surface. And you can also use channel flow-like approaches using viscous numerics. You know, the advantage of this theory is simplicity. When we lose simplicity, we should go to CFD approaches.

GC

I definitely agree. This was just to know if it was possible to completely neglect viscous effects, because although it is only a problem of aspect ratios, the boundary layer thickness depends on the chord length.

[2] Equation (41). The Editor

KVR

Usually I would say that the boundary layer thickness is still much smaller than the real clearance. The influence upon the lifting properties is not significant. Of course the drag has a viscous component, but I would say, and this is known from people who are using finite wings that the largest component of the drag is the induced drag. If the wings are sufficiently thin !

Chairman Grégoire Casalis (GC), SUPAERO-ONERA

The other part of my question was about separation. Do you know if it may occur and where ?

KVR

It happens at the leading edge.

GC

At the leading edge !

KVR

Of course separation effects are much more pronounced. And for example, using matched asymptotics, it is very easy, entering the leading edge, to calculate the minimum pressure point at this leading edge and to use a simple boundary layer calculation in the vicinity of the parabolic leading edge. And our estimates, that are quite easy to obtain as the near field description of the pressure around the leading edge is very simple, give by a simple integration the boundary layer and thus we can find the separation. We can find very interesting points : location of the critical point, location of the minimum pressure point and also location of the separation point as a function of our parametres. It shows that the smaller the altitude, the larger the aspect ratio for a given height, the earlier occurs the separation at the leading edge.

Graham Taylor (GT), Independent Consultant

I wondered if you could just go a little bit over that business about the moving of the centre of gravity ? You made the comment that on an aircraft, stability can be improved by moving the centre of gravity forward and until your lecture today I had the feeling that maybe you could do the same with an ekranoplan but obviously not. Can you say more ?

KVR

Of course for an airplane, it is indispensable to have the centre of gravity ahead of the centre in angle of attack[3]. Of course, you can control the margin.

The only thing I wanted to underline is that for the aircraft, once you designed it, even in a proper way from viewpoint of stability, you can still correct your mistakes by moving the centre of gravity. But with a WIG you cannot because once your vehicle is designed, it has already a certain position of the centres of pitch and height -of course for cruise parametres or a given height, etc.

And if the centre in height is behind the centre in pitch, then theoretically and practically, you can have the lack of static stability. So it means that in the first place you have to design it properly and then improve some characteristics like if you want to go from two channels of control into one channel of control. To improve this, you can use the centre of gravity to position it to the centre in height.

So in the first place, without consideration of what is your distribution of weight along the chord or along the vehicle, you have to take care of the aerodynamic configuration.

Bernard Masure (BM), Université d'Orléans

In this demonstration, the centre of pressure plays no role ?

[3]In the case of an airplane, this point is usually called like that. On a Ground Effect machine, the pitch is usually preferred to describe the state of the craft and thus the word used will be *centre in pitch*. The Editor

KVR

There is always the implicit role of the centre of pressure. For a given wing, as I don't want it to go up and down, I have to find a balance for the cruising mode. So I have to put the centre of gravity, if the aerodynamic forces are the only forces, here I neglect the thrust line, etc., at the position of the centre of pressure.

Stéphan Aubin (SA), Euroavia Toulouse

I have a question about flying wings configurations. Because on this type of craft you don't have a stabilizer like on common ground effect machines. How would you act to have a stable craft - of course having set the pitch and height centres in a position as stable as possible using aerodynamics first ?

KVR

I think we'd better refer to the experience of the people who designed and built the craft. I can only say that you can act on the stability by using the aerodynamic configuration or all kinds of flaps. But I think that Dr Sinitsyn could tell a lot more about that. He has a special control device that gives him the possibility of one channel of control and he knows how to build it. Every designer has his own *know-how*. But the principle is here.